Capital budgeting: A data envelopment analysis approach

Farhad Hosseinzadeh Lotfi*
farhad@hosseinzadeh. ir
Department of mathematics, Science and Research branch, Islamic Azad University , Tehran, Iran

Gholamreza Jahansjhahloo
jahanshahloo@yahoo. com
Department of mathematics, Science and Research branch, Islamic Azad University , Tehran, Iran

Mohsen Rostamy malkhlifeh
Mohsen_rostamy@yahoo. com
Department of mathematics, Islamic Azad University, Science and Research branch, Tehran, Iran

Zohreh Moghaddas
Moghaddas. sahar@gmail. com
Department of Electrical, Computer and Biomedical Engineering, Qazvin Branch, Islamic Azad .University, Qazvin, Iran

Abstract

Budget allocation is a common task carried in our daily life. As it seems to be necessary to perform this task in an ideal manner this paper mainly deals with budget allocation problem. The main key feature of this work is that it uses a mathematical programming technique Data Envelopment Analysis (DEA) for formulating a proper model for capital budget allocation. This formulation is based on the efficiency of organizations the required inputs and the expected outputs from performing new projects. Another key feature of this model is that it has the ability to incorporate both type of qualitative and quantitative data to better reveal the reality of situations .

Keywords:

Data envelopment analysis, Budgeting, Mathematical formulation

Corresponding author: Farhad Hosseinzadeh Lotfi, farhad@hosseinzadeh. ir

۱- Introduction

In daily routines it is of great importance to allocate budget in a proper way. As it seems to be necessary to perform this task in an ideal manner this paper mainly deals with budget allocation problem. For this purpose different methods presents in literature, some of them mentioned in this paper. Each of these papers considers different aspect and discuss about the subject. From early, as existed in literature this issue gained attention from researchers, Gordon et al. (۱۹۹۰) , Regine et al. (۱۹۹۵), and James et al. (۱۹۷۹). After on, other researches followed the subject of budget allocation in different aspects, Frank et al. (۲۰۰۶), Uyar et al. (۲۰۱۱), Roper et al. (۲۰۱۲) and Dutta et al. (۲۰۱۲) .

Also, Wong et al. (۲۰۱۰) presents a gradient based algorithm to solve the data collection budget allocation problem in efficiency measurement of DEA. The important issue is that how the budget should be allocated to various attributes. For solving this problem a two stage gradient method presented. In accordance to the presented method and discussion it reveals that it is necessary to allocate the budget properly and the proposed algorithm serves as a potential method in this field. Lozano et al. (۲۰۱۱) proposed DEA models for determining output target levels, input slacks and input reallocations and additional inputs acquisitions under a capital budget. For demonstrating the proposed methods they performed these models onto the Spanish Port Agency. As Zhao (۲۰۱۰) discussed classical developed to consider interval .

Thus Zhao according to this fact after predicting interval value used interval DEA model for forecasting the efficiency of seventeen entities .

Zhang et al. (۲۰۱۱) provided a multinational capital budgeting problem. The aim question this paper answers is that which project(s) should the investor choose. In this method construction costs, annual net operating cash flows, terminal values of the projects as well as the foreign exchange rates as uncertain variables are incorporated to the analysis. For providing this new model the authors used genetic algorithm. As mentioned above allocating capital budgeting can be performed from different aspects. Each of with considers different conditions and are on basis of various techniques. There is no method deals with this problem while considering DEA analysis which is a very powerful tool for performance evaluation .

Thus the aim of this paper is to introduce a new model based on DEA technique for allocating capital budget. As stated in literature DEA is a mathematical programming technique for performance assessment of a set of homogeneous decision making units (DMUs). The model of this mathematical technique is on basis of some fundamental axioms which can consider different types of production technology. As budgeting is a common task in daily routines it is of importance to formulate a model for allocating budget in a proper way. Thus two models, based on DEA technique, introduced for selecting the proper project from among the presented projects from different organizations and a model for maximizing the total output of the system. A great feature of the presented model is to consider both qualitative and quantitative data into analysis .

۲- Data envelopment analysis

Data envelopment analysis (DEA) is a non-parametric approach for measuring relative efficiency of a set of decision making units (DMUs). This technique is based upon mathematical programming. One of the major advantages in DEA analysis is that it is a linear programming method deals with multiple inputs (X) and multiple outputs (Y) with no pre-assumption about data. Note that in this method it is assumed that input and output vectors are considered to be semi positive. In this technique in accordance to the observations an envelope constructed which surrounds all of them and this leads to generate a frontier which is called as production frontier. It also makes it possible to consider different types of technologies. See Figure۱. which shows the production frontier in constant and variable returns to scale forms of technology, for more information about different production technologies see Banker et al. (۲۰۰۴). The most general way to characterize production technology is production possibility set T, which is defined with a set of semi positive (x, y) as:

$$T = \{(x, y) \mid x \geq \sum_{j=1}^{n} \lambda_j x_j, y \leq \sum_{j=1}^{n} \lambda_j y_j, \lambda_j \geq 0, j = 1, \ldots, n\}$$

Those DMUs located onto this frontier is called best practice which performs efficiently, and those which are far away from this frontier is called inefficient. One important concern of managers is policy making and guiding these inefficient units. One great key feature of DEA technique is that it also can be considered as benchmarks tool, as well. According to the constructed production frontier with efficiently performed units and the fact that relative efficiency of each unit derived from the comparison process to this frontier, this frontier can be considered for benchmarks. Thus for inefficient unit a proper unit, located onto this frontier, can be accounted for as a suitable and achievable target. In the above figure both CCR and BCC production frontiers are depicted .

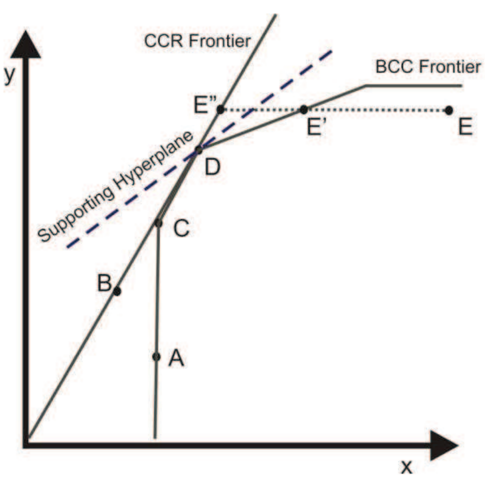

Figure ۱: CCR and BCC frontiers

As it is shown in Figure ۱ a target unit, in input orientation, for DMU$_E$ compared with CCR and BCC frontiers are, respectively, E' and E''. Charnes et al. (۱۹۷۸) presented CCR model for relative efficiency assessment of a set of homogeneous DMUs. In this model constant returns to scale form of technology is assumed. This model is written in input orientation which means inputs are contracted, while keeping the projected point at least at the same output levels. In order to efficiency evaluation and benchmarking one should performed two-stage optimization procedure. This means at first, the optimal value of the objective function of the CCR model is obtained. This scalar shows the possible uttermost radial increment for inputs. Note that simultaneously the shortfall output or excess input usage also needs to be considered .

Thus for finding Pareto- efficient targets, the second stage should be solved. This stage seeks for the non radial improvements for inputs and outputs by maximizing the sum of input and output slacks. Consider this two-stage model as following .

$$Min\ \theta - \varepsilon(۱S^-$$

$$S.t \quad \sum_{j=۱}^{n} \lambda_j x_i$$

$$\sum_{j=۱}^{n} \lambda_j y_۱$$

$$\lambda_j \geq ۰,$$

The same idea can be performed for output orientation which explores higher outputs while keeping the projected point at least at the same input levels. As Banker et al. (۱۹۸۴) presented in a paper, variable returns to scale technology form of the production function can also be considered. In doing so, after performing the analysis constraint $\sum_{j=1}^{n} \lambda_j = 1$ needs to be added to model (۲. ۱). Replacing this constraint with $\sum_{j=1}^{n} \lambda_j \leq 1$ or $\sum_{j=1}^{n} \lambda_j \geq 1$ shows non-increasing and non-decreasing returns to scale technologies, respectively. As mentioned formerly, DEA models are linear programming problem thus it is possible to consider a dual problem for them .

The dual of the above model which is called multiplier form in input orientation is as follows. As stated in literature

$$\sum_{r=1}^{s} u_r y_{rD} - \sum_{i=1}^{m} v_i x_{iD} \leq 1, D \in \{1,..., n\}$$

This inequality forms a supporting hyper plane passing through efficient DMUD. As an instance a supporting hyper plane passing through DMUD is depicted in Figure۱ .

$$Max \quad \sum_{r=۱}^{s} u_r$$

$$S.t. \quad \sum_{i=۱}^{m} v_i$$

$$\sum_{r=۱}^{s} u_r$$

$u_r \geq$

In the above- mentioned model v and u are the input and output weight vectors. Both of the aforesaid models are in input orientation where the input reduction is due to maximized. The above models can be written in output orientation where the output shortfall is due to minimized .

۳. Modeling

In this section the aim is to formulate mathematical programming model based on DEA technique for capital budgeting .

۳. ۱ Project selection formulation

Consider a set of n ministries to be evaluated each of which consumes a vector of m inputs to produce a vector of s outputs, consider Figure ۲ .

Figure ۲: A ministry with input and output

In accordance to the consumed inputs and the produced outputs each ministry, compared to other ministries, has a relative efficiency. As stated in previous section this measure can be obtained via DEA models. The aim is to allocate a capital budget to ministries in a way that the sum of these allocated items does not exceed the entire existing budget. In other words:

$$a_1 + a_2 + \ldots + a_n \leq A$$

Where $a_j,\ j = 1,\ldots,n$ is the allocated budget to the ministry j and A is the rest of total budget needs for performing new projects, as discussed formerly .

Note that it needs to be mentioned that the objective is to allocated budget in a way that it maximizes the total performance. For allocating capital budget it should be considered that for each ministry allocated budget divided into a minimum budget and a surplus budget. Minimum budget utilized in order to fulfill ministry's routine tasks and

Surplus budget needs for coating the inchoate projects and creating new ones. The portion of corresponding budget utilized for creating new project should be allocated to each ministry as regards of

• Efficiency of the ministry in previous years .

• How much input new projects required and what is the output forecasted from this project .

Note that under mentioned circumstances the fundamental issue is that how much allocated budget supported job creation, resolving community problems and creation of income .

An important matter needs to be mentioned is that the forecasted produced can be measured qualitative or quantitative manner, where $\overline{X}_j$ is the consumed input (required budget) of the jth ministry and J is the set of selected projects. Consider the forecasted produced outputs for ministry j as $\overline{Y}_j = (\overline{Y}_j, e_j)^t$ in which e_j is corresponding relative efficiency of that ministry. Another point that makes this method powerful is that this information about the produced output of each ministry can be in either quantitative or qualitative forms. As an instance of qualitative data, fuzzy, interval, stochastic, ordinal and etc. can be mentioned. Rationally from among the proposed project by ministries a subset J should be selected the budget needed for performing these selected projects

• should not exceed the total budget:

$$\sum_{j\in J} \overline{X}j \leq A$$

• The rest of existing budget A cannot be applied to perform another project:

$$\overline{X}j \nleq A - \sum_{j\in J} \overline{X}j, \forall j \notin J$$

Moreover the estimated produced outputs from the selected project j, $j \in J$, for each ministry should:

• Apply in particular macroeconomic conditions .

• Have a high performance .

• Consider limitations upon selected projects which mean:

Number of selected project in ministry j $\leq \alpha_j$, j = ۱. . . , n .

• At least meet minimum requirements .

Now as regards of the above-mentioned discussion two models will be presented for selecting the proper project and allocating capital budget and the other for maximizing the system output .

For formulating a model for project selecting and allocating capital budget consider a binary variable z_j defined as follows:

$$z_j = \begin{cases} ۱, & if\ \text{p} \\ ۰, & \end{cases}$$

Therefore presented model for selecting k project from among the proposed project from ministries is as follows in which Jj, j = ۱,. . . , n is corresponding selected projects from the proposed projects in ministry j, kjis the number of them and the sum all selected projects in n ministries is assumed to be k. Also, as said before the produced outputs should at least satisfy some requirements, in the following model these requirements considered to be $\hat{Y}$.

$$\begin{aligned}
Max\quad & \sum_{j=1}^{k} z_j (U^t \overline{Y}_j - V^t \overline{X}_j) \\
s.t.\quad & z_j (U^t \overline{Y}_j - V^t \overline{X}_j) \le 1, && j = 1,\dots,n, \\
& \sum_{j \in J_j} z_j \le k_j, && j = 1,\dots,n, \\
& \sum_{j=1}^{k} z_j \overline{X}_j \le A, && (3.4) \\
& \overline{X}_j (1 - z_j) \nleq A - \sum_{j=1}^{k} z_j \overline{X}_j, && j = 1,\dots,n, \\
& \sum_{j=1}^{k} \overline{Y}_j z_j \ge \hat{Y}, && j = 1,\dots,n, \\
& U \ge 0, V \ge 0, z_j \in \{0,1\} && j = 1,\dots,n.
\end{aligned}$$

۳. ۲ Maximize total system output with budget allocation

As mentioned formerly a model needed for selecting projects from among the proposed ones that can be satisfy the requiring conditions. After selecting projects the important issue is allocating budget. In what follows a model presented for this subject under the discussed issues .

In order to formulate a model for maximizing total system output first consider the input vector of any ministry j as following:

$$X_j = (X'_j, X''_j)^t$$

Where the first bundles of inputs are those with fixed costs and the others are with variable costs. Consider I' and I'' to be subset of input related to those inputs with fixed and variable costs where $|I'| = m'$ and $|I''| = m''$. Each of these inputs, respectively, has c'_j and c''_j as cost vectors. In this model both j and l are indexes on observed units. The proposed model is as follows:

$$\begin{aligned}
Max \quad & \sum_{r=1}^{s}\sum_{l=1}^{n} \hat{y}_{rl} \\
s.t. \quad & \sum_{j=1}^{n} \lambda_{jl} x_{ij} \le x'_{il}, \qquad i \in I',\ l = 1,\dots,n, \\
& \sum_{j=1}^{n} \lambda_{jl} x_{ij} \le x'_{il}, \qquad i \in I'',\ l = 1,\dots,n, \\
& \sum_{i\in I'}\sum_{j=1}^{n} c'_{ij} x'_{ij} + \sum_{i\in I''}\sum_{j=1}^{n} c''_{ij} x'_{ij} \le A, \qquad (3.5) \\
& \sum_{j=1}^{n} \lambda_{jl} y_{rj} \ge \hat{y}_{rl}, \qquad r = 1,\dots,s,\ l = 1,\dots,n, \\
& \hat{y}_{rl} \ge B_{rl}, \qquad r = 1,\dots,s,\ l = 1,\dots,n, \\
& \hat{y}_{rl} \ge 0,\ x'_{il} \ge 0, \qquad r = 1,\dots,s,\ l = 1,\dots,n,\ i \in I' \cup I''.
\end{aligned}$$

In the above model X' and $\hat{Y}$ are decision variables and B is the minimum requirements on outputs .

٤. Conclusions

As one should properly perform budget allocation in common activities in routine life it felt necessary to have a effective method for it. As mentioned in literature, DEA nowadays utilized as a strong performance assessment tool for performance evaluation technique. A fundamental great feature of this method is that it is mathematical programming technique. Thus incorporating DEA into analysis two models presented for selecting the better projects proposed by organizations or ministries with budget allocation and the other for maximizing the total system output. Each of which is formulated via DEA technique. Note that formulating of these models different aspects of capital budget allocation is taken into consideration to better show the reality of situations .

References

[١] Banker, R. D, Charnes, A, & Cooper, W. W, Some models for estimating technical and scale efficiencies in data envelopment analysis, Management Science, ٣٠, ١٩٨۴, ١٠٧٨–١٠٩٢ .

[٢] Charnes, A. , Cooper, W. W, Rhodes, E, Measuring the efficiency of decision making units, European journal of operational research ٢,١٩٧٨, ۴٢٩–۴۴۴ .

[٣]Gordon, L. A. , Loeb, M. P. Stark, A. W. Capital budgeting and the value of information Original Research Article Management Accounting Research, ١,١٩٩٠, ٢١–٣۵ .

[۴] Gui-sheng, ZHAO, Based on Interval DEA to Forecast Efficiency and Formulate Budget Schedule, International Conference on Management Science and Engineering, ٢٠٠٩, ١٣۴١–١٣۴٧ .

[۵]James C. Wetherbe, Gary W. Dickson. Zero-based budgeting: An alternative to charge out systems Original Research Article Information & Management,٢,١٩٧٩, ٢٠٣–٢١٣ .

[۶] Lozano, Sebastian , Villa, Gabriel , Canca, David, Application of centralized DEA approach to capital budgeting in Spanish ports, Computers and Industrial Engineering, ۶٠, ٢٠١١ ,۴۵۵–۴۶۵ .

[٧]Roper, Andrew H. , Ruckes, Martin E. Intertemporal capital budgeting Original Research Article Journal of Banking & Finance, ٣۶, ٢٠١٢, ٢۵۴٣–٢۵۵١ .

[٨] Rajiv, D, Banker, William, W, Cooper, Lawrence, M, Seiford, Robert M, Thrall, Joe Zhu, Returns to scale in different DEA models, European Journal of Operational Research, ١۵۴, ٢٠٠۴, ٣۴۵٣۶٢ .

[٩]Regine Slagmulder, Werner Bruggeman, Luk van Wassenhove. An empirical study of capital budgeting practices for strategic investments in CIM technologies Original Research ArticleInternational Journal of Production Economics, ۴٠,١٩٩۵, ١٢١–١۵٢ .

[١٠]Dutta, Sunil, Fan, Qintao. Incentives for innovation and centralized versus delegated capital budgeting Original Research ArticleJournal of Accounting and Economics, ۵٣, ٢٠١٢, ۵٩٢–۶١١ .

[١١] Zhang, Qun , Huang, Xiaoxia, Tang, Leming, Optimal multinational capital budgeting under uncertainty, Computers and Mathematics with Applications ۶٢ ,٢٠١١, ۴۵۵٧–۴۵۶٧ .

[۱۲]Verbeeten Frank H. M. . Do organizations adopt sophisticated capital budgeting practices to deal with uncertainty in the investment decision?: A research note Original Research ArticleManagement Accounting Research, ۱۷, ۲۰۰۶, ۱۰۶–۱۲۰ .

[۱۳]Uyar, Ali, Bilgin, Necdet. Budgeting practices in the Turkish hospitality industry: An exploratory survey in the Antalya region Original Research ArticleInternational Journal of Hospitality Management, ۳۰, ۲۰۱۱, ۳۹۸–۴۰۸ .

[۱۴] Wong, Wai Peng , Lee, Loo Hay, Jaruphongsa, Wikrom, A Two-phased Gradient Technique for Budget Allocation: An Application to Data CollectionBudget Allocation in Efficiency Measurement of DEA, ICECT, ۲۰۱۰, ۹۳–۹۶ .